**Section 1**

**10 marks**
**Attempt Questions 1 -10**
**Allow about 15 mins for this section**

Use multiple choice answer sheet for Questions 1 -10

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**1.** $\sin 3\alpha \cos 3\alpha =$

A) $\frac{1}{2}\sin 6\alpha$ B) $9\sin\alpha \cos\alpha$ C) $\frac{1}{2}\sin 6\alpha$ D) $2\sin 2\alpha$

**2.** The point R divides the interval joining P $(-3, 6)$ and Q $(6, -6)$ externally in the ratio 2:1.

Which of these are coordinates of R?

A) $(3, -2)$ B) $(15, -18)$ C) $(0, 2)$ D) $(-12, 18)$

**3.** The polynomial $3x^3 - 4x^2 + 2x - 1 = 0$ has roots $\alpha$, $\beta$, and $\gamma$.

What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

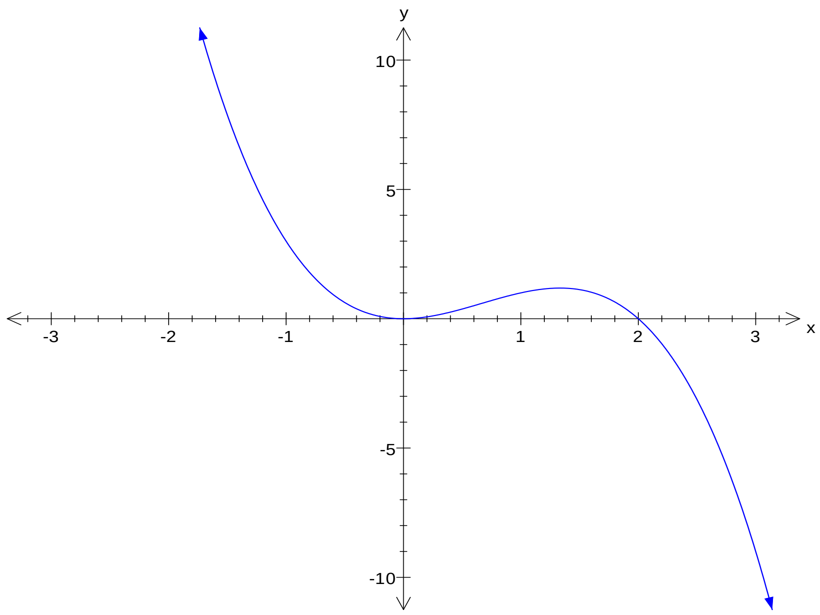
A) $-\frac{1}{2}$ B) $2$ C) $-2$ D) $-4$

**4.** Which of the following are solutions of $\frac{x}{2x-1} \geq 0$ ?

A) $x \geq 0$ B) $0 \leq x < \frac{1}{2}$

C) $x \geq -\frac{1}{2}$ D) $x \leq 0, x > \frac{1}{2}$

**5.** Which of the following could be the equation of the polynomial shown below?




A) $P(x) = x^2(2 - x)$ B) $P(x) = x(2 - x)$

C) $P(x) = (x - 2)x^2$ D) $P(x) = x(x - 2)$

**6.** What is the derivative of $y = \tan^{-1}\frac{1}{3x}$?

A) $\frac{9x^2}{1+9x^2}$ B) $\frac{3x}{3+x^2}$ C) $-\frac{3x}{1+9x^2}$ D) $-\frac{3}{9x^2+1}$

**7.** A particle is moving in simple harmonic motion with displacement x. Its velocity v is given by $v^2 = \frac{1}{4}(16 - x^2)$. What are the amplitude and period of the motion?

A) amplitude 4, period $4\pi$ B) amplitude 16, period $\frac{1}{2\pi}$

C) amplitude 4, period $\frac{\pi}{4}$ D) amplitude $\frac{1}{2}$, period $\frac{\pi}{4}$

**8.** $\int sin^2 x dx =$

A) $\frac{1}{2}x - \frac{1}{4}cos2x + c$

B) $\frac{1}{2}x - \frac{1}{4}sin2x + c$

C) $\frac{1}{2}x + \frac{1}{4}cos2x + c$

D) $\frac{1}{2}x + \frac{1}{4}sin2x + c$

**9.** Evaluate $\lim_{x \to 0} \frac{\sin \frac{x}{3}}{2x}$

A) $\frac{2}{3}$ B) $\frac{1}{6}$ C) $\frac{3}{2}$ D) 6

**10.** What is the coefficient of $x^4$ in the expansion of $(1 - x)(1 + x)^9$?

A) 9 B) -84 C) 42 D) 126

# Section II

**60 Marks**
**Attempt Questions 11 – 14**
**Allow 1 hours 45 minutes for this section.**

Answer the questions in the own writing booklets provided.

Start each question on a new page.

All necessary working should be shown in every question

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**Question 11** (15 marks) **Marks**

(a) Use the substitution $t = tan\frac{x}{2}$ to show that $\frac{1+cosx+sinx}{1-cosx+sinx} = cot\frac{x}{2}$ **3**

(b) Use the substitution $u = 5 - x$, to find the exact value of $\int_1^5 x\sqrt{5-x}dx$ **3**

(c) Let $f(x) = \frac{1}{\sqrt{x-1}}$

(i) Find the domain and range of $f(x)$. **2**

(ii) Find an expression for the inverse function $f^{-1}(x)$. **2**

(iii) Find the domain and range of the inverse function. **1**

(d) Prove by mathematical induction that,

$$\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)\times \dots\dots\dots \left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$$ for all integer $n \geq 2$ **4**

**Question 12** (15 marks) Start this question in a new booklet. **Marks**

(a) A metal rod is taken from a freezer at $-10^oC$ into a room where the air temperature is $25^oC$.

The rate at which the rod warms follows Newton's law, that is

$$\frac{dT}{dt} = -k(T - 25)$$

Where k is a positive integer, time t is measured in minutes and temperature *T* is measured in degrees Celcius.

(i) Show that $T = 25 - Ae^{-kt}$ is a solution of the differential equation $\frac{dT}{dt} = -k(T - 25)$, and find the value of A. **2**

(ii) If the temperature of the rod reaches $5^oC$ in 80 minutes, find the exact value of k. **2**

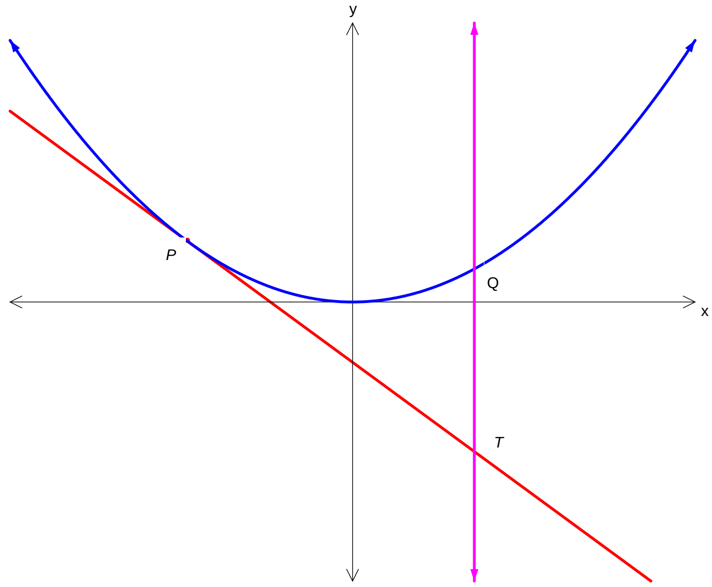
(iii) Find the temperature of the rod after a further hour. **1**

(iv) Sketch the graph of the temperature of the rod against time. **1**

*Question 12 continues on the next page.*

**Question 12 continued** **Marks**

(b) The points $P(6p, 3p^2)$ and $Q(6q, 3q^2)$ lie on the parabola $x^2 = 12y$.




(i) Show that the equation of the tangent to the parabola at *P* is $y = px - 3p^2$ **2**

(ii) The tangent at *P* and the line passing through *Q* parallel to the y-axis Intersect at *T*. Show that the coordinates of *T* are $(6q, 6pq - 3p^2)$. **2**

(iii) Find the coordinates of *M*, the midpoint of *PT*. **1**

(iv) Find the Cartesian equation of the locus of M when $pq = -2$. **1**

(c) (i) Show that the equation $lnx - cosx = 0$ has a root between $x = 1$ and $x = 2$. **1**

(ii) By taking $x = 1.2$ as the first approximation, use one step of Newton's method to find a better approximation to this root. Answer to 2 decimal places. **2**

**Question 13** (15 marks) Start this question in a new booklet. **Marks**

(a) Sketch the function $y = \pi\cos^{-1}\frac{x}{2}$ **2**

(b) The equation of motion for a particle undergoing simple harmonic motion is

$$\ddot{x} = -n^2x$$

Where x (metres) is the displacement of the particle from the origin at time t (seconds) and n is a positive constant.

(i) Verify that $x = asin(nt + \alpha)$, where *n*, *t*, and *α* are constants, is a solution of the equation of motion. **1**

(ii) The particle is initially stationary at $x = 2$ and the function has a period of 3 seconds, find the values of *n*, *a*, and *α.* **3**

(c) Consider the function $f(x) = xe^{x^2}$

(i) Show that the gradient of this function is always positive. **2**

(ii) Find any points of inflexion. **1**

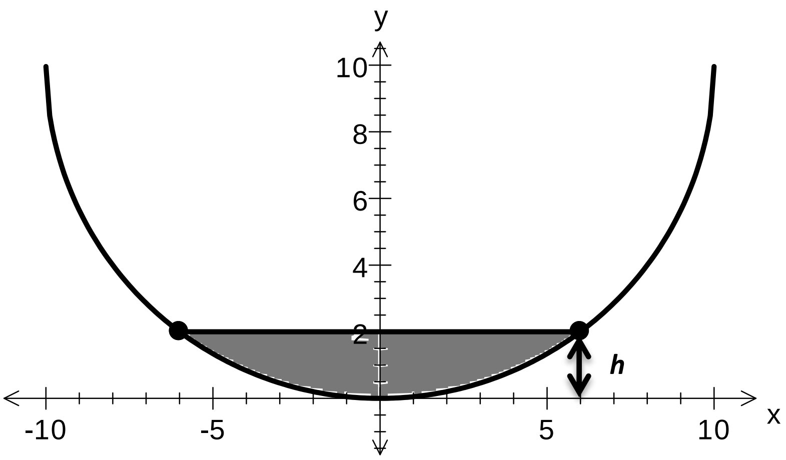
(iii) Sketch the graph of $y = f(x)$ **1**

*Question 13 continues on the next page.*

**Question 13 continued** **Marks**

(d) A hemispherical bowl of radius 10cm is being filled with water at a constant rate of 25 $cm^3$ per minute.

(i) By finding the volume of revolution formed by rotating the semi circle $y = 10 - \sqrt{100 - x^2}$ about the **y-axis** from $y = 0$ to $y = h$, show that the volume ($V$) of the water in terms of its depth ($h$) is given by $V = \pi\left(\frac{-h^3}{3} + 10h^2\right)$. **3**



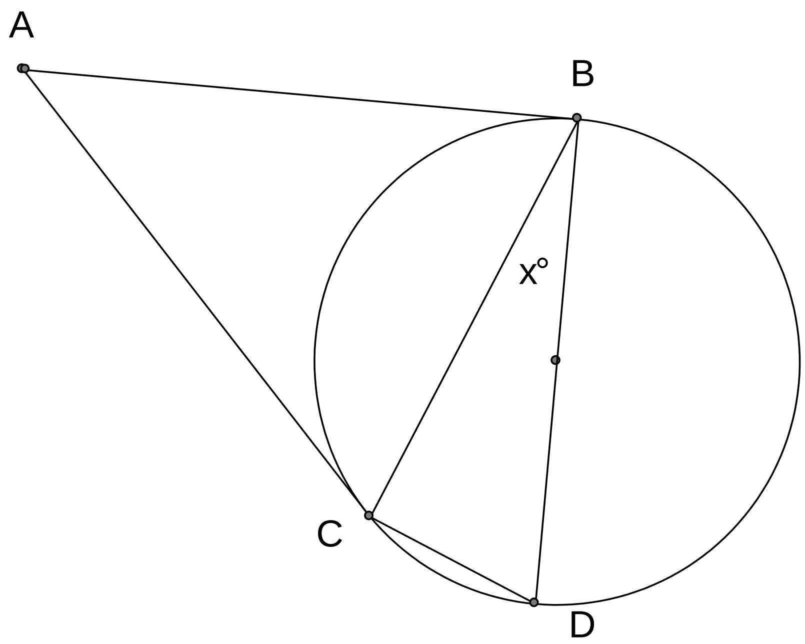

(ii) At what rate is the water rising when the depth is 5cm. Answer to 2 decimal places. **2**

**Question 14** *(15 marks)* Start this question in a new booklet.

(a) AB and AC are tangents to a circle. D is a point on the circle such that $< BDC =< BAC$ and $\angle BAC = 2\times\angle DBC$. Let $< DBC = x$.

(i) Show that *DB* is a diameter. **4**

(ii) Show that $BC = AB$. **1**




(b) Consider the expansion:

$$(1+x)^{2n+1} = \binom{2n+1}{0} + \binom{2n+1}{1}x + \binom{2n+1}{2}x^2 + \ldots\ldots\ldots \binom{2n+1}{2n+1}x^{2n+1}$$

(i) Show that $\sum_{r=1}^{2n+1}\binom{2n+1}{r} = 2\times 4^n - 1$ **2**

(ii) By differentiating, the original expansion above with respect to *x*, or otherwise, show that

$$\binom{2n+1}{1} - 2\binom{2n+1}{2} + 3\binom{2n+1}{3} - \ldots.+(2n+1)\binom{2n+1}{2n+1} = 0$$ **3**

*Question 14 continues on the next page.*

**Question 14 continued** **Marks**

(c) A cannonball is fired from a cannon on top of an 80 metre high cliff down on to a ship in the sea below with velocity 40 metres per second at an angle of inclination θ to the horizontal. The equations of motion of the cannonball are:

$x = 35tcos\theta$ and $y = -4.9t^2 + 35tsin\theta + 80$ (Do NOT prove this).

(i) By eliminating $t$ show that the Cartesian equation of the path of the cannonball is given by $y = \frac{-sec^2\theta}{250}x^2 + xtan\theta + 80$ **2**

(ii) In order to hit the ship, the cannonball must land 150 metres from the base of the cliff. For what values of θ will this occur? **3**

## End of Paper

MC

Q1 $\sin 2A = 2\sin A \cos A$.

let $A = 3\alpha$ $\quad \sin 6\alpha = 2\sin 3\alpha \cos 3\alpha$.

$\sin 3\alpha \cos 3\alpha = \frac{1}{2}\sin 6\alpha$ (A)

Q2. Use ratio $2 : -1$

$$R = \left(\frac{2\times 6 - 1\times -3}{2-1}, \frac{2\times -6 - 1\times 6}{2-1}\right)$$

$= (15, -18)$ (B)

Q3. $\alpha+\beta+\gamma = \frac{4}{3}$

$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{2}{3}$

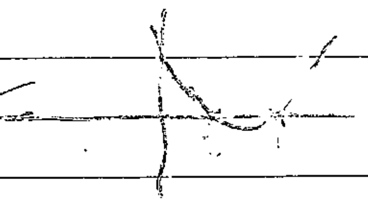
$\alpha\beta\gamma = \frac{1}{3}$

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma}$$

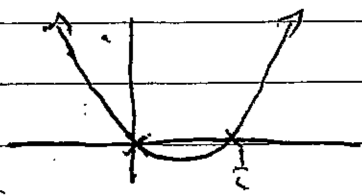
$$= \frac{\frac{2}{3}}{\frac{1}{3}}$$

$= 2$. (B)

Q4.



$x(2x-1) \geq 0$. $\quad x \neq \frac{1}{2}$



$x \leq 0, x > \frac{1}{2}$. (D)

Q5. (A).

6. $y = \tan^{-1}\frac{1}{3x}$

$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{3x}\right)^2} \times -\frac{1}{3}x^{-2}$

$= \frac{1}{1+\frac{1}{9x^2}} \times -\frac{1}{3x^2}$

$= -\frac{1}{3x^2+\frac{1}{3}} \frac{\times 3}{3} = -\frac{3}{9x^2+1}$ (D)

Q7. $v^2 = n^2(a^2 - x^2)$  $a = 4$  $n = \frac{1}{2}$

amplitude ... 4

period $= \frac{2\pi}{n}$

$= 4\pi$ (A)

Q8. (B)

Q9 $\frac{1}{3} \times \frac{1}{2} \lim_{x \to 0} \frac{\sin\frac{x}{3}}{\frac{x}{3}} = \frac{1}{6} \times 1$

$= \frac{1}{6}$ (B)

Q10 $(1-x)\left({}^9C_0 + {}^9C_1 x + {}^9C_2 x^2 + {}^9C_3 x^3 + {}^9C_4 x^4 + \dots\right)$

Coeff of $x^4$: $1 \times {}^9C_4 - 1 \times {}^9C_3$

$= 126 - 84$ (C)

Q11(a) $t = \tan\frac{x}{2}$ $\therefore \sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

$$LHS = \frac{1 + \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}}{1 - \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}}$$ ✓

$$= \frac{\frac{1+t^2+1-t^2+2t}{(1+t^2)}}{\frac{1+t^2-(1-t^2)+2t}{(1+t^2)}}$$

$$= \frac{2+2t}{2t+2t^2}$$ ✓

$$= \frac{2(1+t)}{2t(1+t)}$$

$$= \frac{1}{t}$$ ✓

$$= \cot\frac{x}{2}$$

(b) $\int_1^5 x\sqrt{5-x}\,dx$

$u = 5 - x$ $x=1$ $u=4$

$x = 5 - u$ $x=5$ $u=0$

$\frac{dx}{du} = -1$

$dx = -du$

$$= \int_4^0 (5-u)\,u^{\frac{1}{2}} \times -du$$ ✓

$$= \int_0^4 5u^{\frac{1}{2}} - u^{3/2}\,du$$ ✓

$$= \left[\frac{10}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^4$$ ✓

$$= \left(\frac{10}{3}\times 8 - \frac{2}{5}\times 32\right) - 0$$

$$= 13\frac{13}{15} \quad \left(\frac{208}{15}\right)$$ ✓

(b)

(i) $y = \frac{1}{12}x^2$

$\frac{dy}{dx} = \frac{1}{6}x$

At $x = 6p$ $\frac{dy}{dx} = p$ ✓

∴ Equation of tangent is

$y - 3p^2 = p(x - 6p)$ ✓

$y - 3p^2 = px - 6p^2$

$y = px - 3p^2$ ①

(ii) Line through Q parallel to y-axis is $x = 6q$. ② ✓

Point T is where ① & ② meet

ie $y = p \times 6q - 3p^2$ ✓

$y = 6pq - 3p^2$

∴ intersection is at $T(6q, 6pq - 3p^2)$

(iii) $M = \left(\frac{6p+6q}{2}, \frac{3p^2+6pq-3p^2}{2}\right)$

$= (3p+3q, 3pq)$ ✓

(iv) when $pq = -2$, y-value of M is $-6$ ie constant

∴ equation of locus of M is $y = -6$ ✓

(c)(i) let $f(x) = \ln x - \cos x$

$f(1) = -0.5403...$

$f(2) = 1.10929....$ ✓

since $f(1) < 0$, $f(2) > 0$ and $f(x)$ is continuous in the interval, there must be a root.

(ii) $f'(x) = \frac{1}{x} + \sin x$ ✓

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 1.2 - \frac{\ln 1.2 - \cos 1.2}{\frac{1}{1.2} + \sin 1.2}$ ✓

$= 1.30$.

(c). $f(x) = \frac{1}{\sqrt{x-1}}$ ✓

(i) domain $x > 1$ ✓

range $y > 0$ ✓

(ii) let $y = \frac{1}{\sqrt{x-1}}$

swap $x, y$ for inverse

$x = \frac{1}{\sqrt{y-1}}$ ✓

$\sqrt{y-1} = \frac{1}{x}$

$y-1 = \frac{1}{x^2} \quad x > 0$

$y = \frac{1}{x^2} + 1 \ ; \ x > 0$ ✓ must have domain

(iii) ~~domain $x > 0$~~

range $y > 1$ ✓

(d) Show true for $n = 2$

$LHS = 1 - \frac{1}{2^2} = \frac{3}{4}$ $\quad RHS = \frac{2+1}{4} = \frac{3}{4}$ ✓

$LHS = RHS$

If true for $n = k$

$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \times \ldots \times \left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k}$ *

Show true for $n = k+1$

ie $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \ldots \left(1 - \frac{1}{k^2}\right)\left(1 - \frac{1}{(k+1)^2}\right) = \frac{k+2}{2k+2}$ ✓

$LHS = \left(\frac{k+1}{2k}\right)\left(1 - \frac{1}{(k+1)^2}\right)$ from *. ✓

$= \left(\frac{k+1}{2k}\right)\left(\frac{(k+1)^2 - 1}{(k+1)^2}\right)$

$= \left(\frac{k+1}{2k}\right)\left(\frac{k^2+2k}{(k+1)^2}\right)$

$= \frac{(k+1)}{2k} \times \frac{k(k+2)}{(k+1)^2}$ ✓

$= \frac{k+2}{2k+2} = RHS$

correct
(-1 if no ~~statement~~)

$\therefore$ true for all $n \geq 2$ by Principle of Mathematical Induction

Q 12(a) (i) $T = 20 - Ae^{-kt}$

$\frac{dT}{dt} = kAe^{-kt}$ | but $Ae^{-kt} = -(T-20)$

$= -k(T-20)$ ✓

when $t = 0$ $T = -10$.

$-10 = 20 - Ae^{0}$

$A = 30$. ✓

$\therefore T = 20 - 30e^{-kt}$

(ii) when $t = 80$ $T = 5$.

$5 = 20 - 30e^{-80k}$ ✓

$30e^{-80k} = 15$.

$e^{-80k} = \frac{1}{2}$.

$-80k = \ln\frac{1}{2}$.

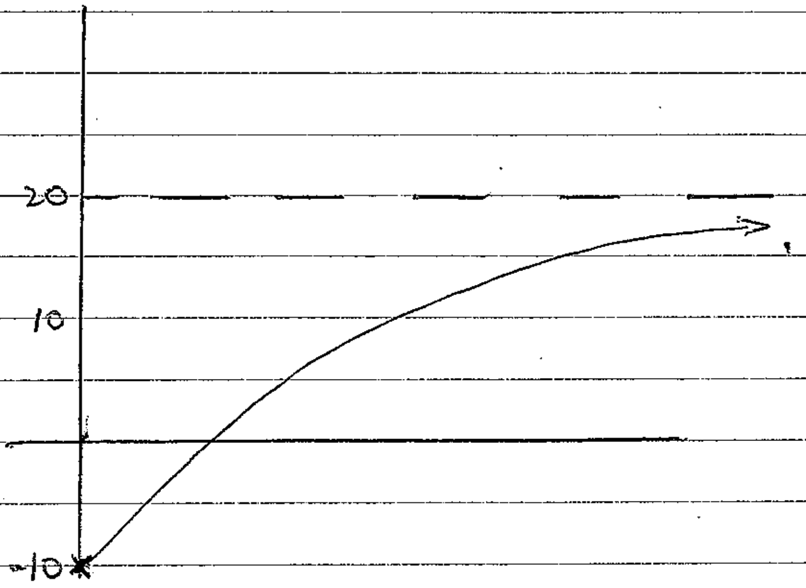
$k = \frac{\ln\frac{1}{2}}{-80}$. ✓

(iii) when $t = 80 + 60$

$= 140$.

$T = 20 - 30e^{-k \times 140}$. ✓

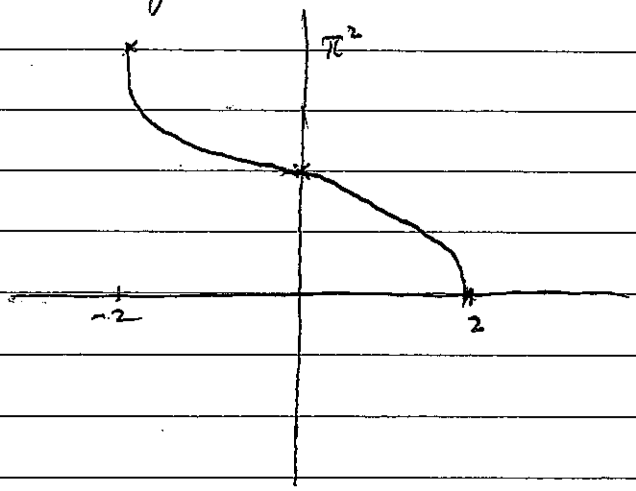
$T = 11$



✓

Q13(a) domain : $-1 \le \frac{x}{2} \le 1$ $\quad -2 \le x \le 2$.

range : $0 \le \frac{y}{\pi} \le \pi$ $\quad 0 \le y \le \pi^2$




(b)(i) $x = a\sin(nt+\alpha)$

$\dot{x} = na\cos(nt+\alpha)$

$\ddot{x} = -n^2 a\sin(nt+\alpha)$

$= -n^2 x$

(ii) $t=0$, $\dot{x}=0$, $x=2$

Initially stationary at $x=2$ means $a=2$

and $2 = 2\sin(0+\alpha)$

$\sin\alpha = 1$

$\alpha = \pi/2$

period = 3 seconds

$\frac{2\pi}{n} = 3$

$n = \frac{2\pi}{3}$

(c) $f'(x) = x \times 2xe^{x^2} + e^{x^2}$

$= e^{x^2}(2x^2+1)$

since $e^{x^2} > 0$ and $2x^2 > 0$ $\quad f'(x) > 0$ for all $x$

$f''(x) = 2x^2 \times [2xe^{x^2}] + e^{x^2} \times 4x + 2xe^{x^2}$

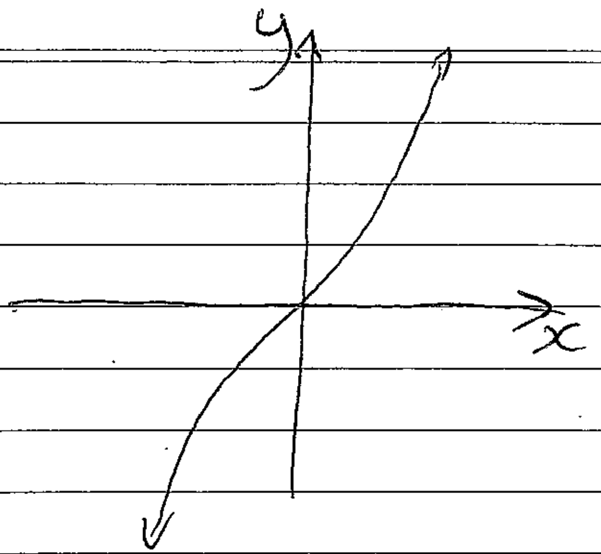
$= 4x^3e^{x^2} + 6xe^{x^2}$

$= 2xe^{x^2}(2x^2+3)$

$f''(x) = 0$ when $x=0$ $\quad$ (note $e^{x^2} > 0$, $2x^2+3 > 0$)

At $x=0$ $f'(0) = 1 > 0$.

$\therefore$ inflexion at $(0,0)$.




d)(i) $y = 10 - \sqrt{100 - x^2}$

$\sqrt{100 - x^2} = 10 - y$

$100 - x^2 = (10 - y)^2$

$x^2 = 100 - (10 - y)^2$

$x^2 = 100 - (100 - 20y + y^2)$

$x^2 = 20y - y^2$

$Vol = \pi \int_0^h x^2 \, dy$

$= \pi \int_0^h 20y - y^2 \, dy$

$= \pi \left[ 10y^2 - \frac{y^3}{3} \right]_0^h$

$V = \pi \left( 10h^2 - \frac{h^3}{3} \right)$

(ii) $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $\qquad \frac{dV}{dh} = \pi (20h - h^2)$

$25 = \pi (20h - h^2) \times \frac{dh}{dt}$

at $h = 5$

$25 = \pi (100 - 25) \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{3\pi}$ cm/sec

Q4(a)

(i) if $D\hat{B}C = x$, then $B\hat{A}C = 2x$ (given)

$AB = AC$ (tangents from external point are equal)

$\therefore \triangle ABC$ is isosceles

and $A\hat{B}C = A\hat{C}B$ (base < isos $\triangle$)

$\therefore A\hat{B}C = \frac{180-2x}{2}$ ( < sum $\triangle ABC$)

$= 90 - x$

$\therefore A\hat{B}D = 90 - x + x$

$= 90°$

$\therefore$ BD is perpendicular to tangent AB and must pass through centre of circle

so BD is a diameter.

(ii) if $B\hat{D}C = B\hat{A}C$

and $B\hat{D}C = 90 - x$ (alternate segment theorem)

then $2x = 90 - x$

$3x = 90$

$x = 30$

$\therefore C\hat{A}B = A\hat{B}C = B\hat{C}A = 60°$

$\therefore \triangle ABC$ is equilateral

and $BC = AB$

(b)(i) sub $x = 1$

$2^{2n+1} = \binom{2n+1}{0} + \binom{2n+1}{1} + \binom{2n+1}{2} + \dots + \binom{2n+1}{2n+1}$

$2 \times 4^n = 1 + \binom{2n+1}{1} + \binom{2n+2}{2} + \dots + \binom{2n+1}{2n+1}$

$2 \times 4^n - 1 = \binom{2n+1}{1} + \binom{2n+1}{2} + \dots + \binom{2n+1}{2n+1}$

$= \sum_{r=1}^{2n+1} \binom{2n+1}{r}$

by differentiating both sides

$(2n+1)(1+x)^{2n} = \binom{2n+1}{1} + 2\binom{2n+1}{2}x + 3\binom{2n+1}{3}x^2 + \dots + (2n+1)\binom{2n+1}{2n+1}x$

sub $x = -1$

$0 = \binom{2n+1}{1} - 2\binom{2n+1}{2} + 3\binom{2n+1}{3} - \dots + (2n+1)\binom{2n+1}{2n+1}$.

(c) $x = 35t\cos\theta$ ① $\quad y = -4.9t^2 + 35t\sin\theta + 80$ ②

From ① $t = \dfrac{x}{35\cos\theta}$

sub in ② $y = -4.9\left(\dfrac{x}{35\cos\theta}\right)^2 + 35\left(\dfrac{x}{35\cos\theta}\right)\sin\theta + 80.$

$$y = -\frac{4.9}{1225}x^2\sec^2\theta + x\tan\theta + 80$$

$$y = -\frac{\sec^2\theta}{250}x^2 + x\tan\theta + 80.$$

(ii) when $y = 0$ $x = 150.$

$$0 = -90\sec^2\theta + 150\tan\theta + 80.$$

$$0 = -90.(\tan^2\theta + 1) + 150\tan\theta + 80.$$

$$0 = -90\tan^2\theta - 90 + 150\tan\theta + 80$$

$$0 = -90\tan^2\theta + 150\tan\theta - 10.$$

$$9\tan^2\theta - 15\tan\theta + 1 = 0.$$

$$\tan\theta = \frac{15 \pm \sqrt{15^2 - 4\times 9\times 1}}{18}.$$

$\tan\theta = 1.597\ldots$ or $0.0695\ldots$

$\theta = 58^\circ$ or $4^\circ$